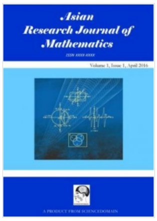

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Modeling the Effect of Friction on Quadratic Depth Profile

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In the inviscid rip-current regime it was demonstrated that there is a longshore component of the rip current system, which does not need friction in order to exist. Hence here in the viscous regime we are not concerned with any frictionally determined currents, but only with how the frictional terms modify the already constructed inviscid solution. Note that the frictional terms are not invoked in the shoaling zone, and so the solution remains unchanged in that regime. The nearshore is characterized by the presence of breaking waves, and so we developed equation to be used outside surf zone, based on small-amplitude wave theory, and another set of equations to be used inside the surf zone, based on an empirical representation of breaking waves, Suitable matching conditions are applied at the boundary between the offshore shoaling zone and the nearshore surf zone. Both sets of equation are obtained by averaging the basic equations over the wave phase. Thus the qualitative solution constructed is a free vortex defined in both shoaling and surf zone where in the surf zone the free vortex is perturbed by a long shore component.

Keywords: Wave-current interactions; setup; set down; surf zone; shoaling zone; longshore currents.

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1 Introduction

The hydrodynamics of the nearshore zone constitutes an important area of studies. The interest is mainly driven by a combination of engineering, shipping and coastal interests. There has been much research on shoaling nonlinear waves, on how currents affect waves and how waves can drive currents. The basis for this subject was laid down by [1], [2], were analysis was done for nonlinear interaction between short waves and long waves (or currents), and showed that variations in the energy of the short waves correspond to work done by the long waves against the *radiation stress* of the short waves. In the shoaling zone this radiation stress leads to what are now known as wave setup and wave setdown, surf beats, the generation of smaller waves by longer waves, and the steepening of waves on adverse currents, [3], [4]. The divergence of the radiation stress was shown to generate an alongshore current by obliquely incident waves on a beach [5], [6]. During the shoaling of the waves there is a discontinuity in the wave energy in the mean vorticity equation of the waves. Thus there is sharp distinction in the behaviour of waves in the regimes due largely to forcing.

As wave groups propagate towards the shore, they enter shallower water and eventually break on beaches. The important process here is the wave breaking and dissipation of energy. The focusing of energy and the wave height variation across the group forces low frequency long waves that propagate with the group velocity [1]. These long waves may be amplified by continued forcing during the shoaling of the short wave group into shallower water [3], [7], [8] and [9]. In sufficiently shallow water, the short waves within the group may break at different depths leading to further long wave forcing by the varying breaker-line position [10], [11]. This means that the shoreward propagating waves may reflect at the shoreline and subsequently propagate offshore [12].

Wave breaking leads to a transfer of the incoming wave energy to a range of different scales of motions, and particularly to lower frequencies [13]. Thus waves called surf beat [12]; [14], may propagate in the cross-shore direction (called leaky waves). Waves may be trapped refractively as edge waves [15]. Wave breaking may occur for two reasons. Firstly due to natural variation in the wave direction and amplitude. These changes occur in space and time. Secondly wave may break due to topographic influences. When this is the case, as in the nearshore zone, the location and form of the wave breaking is influenced by the bottom depth profile.

Essentially we derived a model for the interaction between waves and currents. The aim is to provide analytical solution for waves in the nearshore zone on time scales longer than an individual wave. This is possible on long-time scales using the wave-averaging procedure often employed in the literature [16]. We describe solutions for rip currents, in the shoaling zone matched to the surf zone, for two different beach profiles.

The structure of the mathematical model is based on the Euler equations for an inviscid incompressible fluid. We then employ an averaging over the phase of the waves, exploiting the difference in time scales between the waves and the mean flow, which is our main interest. The nearshore zone is divided into regions, a shoaling zone where the wave field can be described by linear sinusoidal waves, and the surf zone, where the breaking waves are modelled empirically. The breakerline is fixed at $x = x_b$ but in general could vary.

In the shoaling zone wave field, we use an equation set consisting of a wave action equation, combined with the local dispersion relation and the wave kinematic equation for conservation of waves. The mean flow field is then obtained from a conservation of mass equation for the mean flow, and a momentum equation for the mean flow driven by the wave radiation stress tensor. In the surf zone, we use a standard empirical formula for the breaking wave field, together with the same mean flow equations.

2 Basic Governing Equations

The basic set of equations used in this paper are

$$\omega^2 = g\,\kappa \tanh \kappa\, h \tag{2.1}$$

$$\mathbf{k_t} + \nabla\omega = 0\,, \tag{2.2}$$

$$E_t + \nabla\cdot(\mathbf{c}_g E) = 0\,, \tag{2.3}$$

that is (i) dispersion relation, (ii) conservation of waves equation and (iii) wave action equation. The second set of the basic equations are

$$\frac{\partial\bar{\zeta}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{H}U_i) = 0 \tag{2.4}$$

$$\bar{H}\frac{\partial U_i}{\partial t} + \bar{H}U_j\frac{\partial U_i}{\partial x_j} \;=\; -\frac{\partial}{\partial x_j}\,S_{i\,j} \;-g(\bar{\zeta}+h)\frac{\partial\bar{\zeta}}{\partial x_j} \tag{2.5}$$

that is, the mass conservation equation and mean momentum equation respectively where the **radiation stress** acts as the driving force of the waves define as follows:

$$S_{i\,j} \;=\; c_{gi}k_j\frac{E}{\omega} + \delta_{i\,j}E[\frac{c_g}{c} - \frac{1}{2}]\,. \tag{2.6}$$

These equations have been derived outside the surf zone. Inside the surf zone we also use the mean mass and momentum equations but follow the conventional literature and replace the radiation stress tensor by an empirically determined quantity [16].

Previously in our work elsewhere we constructed the longshore current driven by the radiation stress in the surf zone when this forcing has no alongshore modulation, that is there is no y-dependence in the radiation stress. In that case the frictional effects are necessary for the longshore current to exist, and the weaker the friction the stronger is the current. Also in the inviscid rip-current model constructed see [17], we have demonstrated that there is a longshore component of the rip current system, namely $Z(x)$ [17] which does not need friction in order to exist. Hence here we are not concerned with any frictionally determined currents, but only with how the frictional terms might modify the already constructed inviscid solution. Before proceeding note that the friction terms are not invoked in the shoaling zone, and so the solution there remains unchanged.

2.1 Fundamentals of friction in the nearshore

The full momentum equations with the frictional terms included are in [16]

$$H[U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y}] = -g\,H\frac{\partial\zeta}{\partial x} - [\tau_x] + \tau'_x \tag{2.7a}$$

$$H[U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y}] = -g\,H\frac{\partial\zeta}{\partial y} - [\tau_y] + \tau'_y\,. \tag{2.7b}$$

Here the radiation stress terms are given by [2.6],

$$\tau_x = \frac{\partial S_{11}}{\partial x} + \frac{\partial S_{12}}{\partial y} \quad \text{and } \tau_y = \frac{\partial S_{21}}{\partial x} + \frac{\partial S_{22}}{\partial y}\,. \tag{2.8}$$

$$\tag{2.9}$$

The terms τ'_y and τ'_x are lateral mixing terms which describe the afore-mentioned frictional effects, and are expressed as follows in [18] and [16]

$$\tau'_y = \nu_e v_{xx}, \tau'_x = \nu_e u_{xx} \quad \text{where} \quad \nu_e = \nu_0 g^{1/2}h^{3/2}\,. \tag{2.10}$$

Next we recall that the steady-state mean mass equation (2.4) can be solved using a transport stream function $\psi(x, y)$ in the expressions so that

$$U = -\frac{1}{H}\frac{\partial \psi}{\partial y} \quad \text{and } V = \frac{1}{H}\frac{\partial \psi}{\partial x}. \tag{2.11}$$

Next, again eliminating the pressure, we obtain the mean vorticity equation in the frictional regime

$$\psi_x(\frac{\Omega}{H})_y - \psi_y(\frac{\Omega}{H})_x = [\frac{\tau_x}{H}]_y - [\frac{\tau_y}{H}]_x + \mu_0[[h^{\frac{3}{2}}(\frac{\psi_x}{h})_{xx}]_x + [h^{\frac{3}{2}}(\frac{\psi_y}{h})_{xx}]_y], \tag{2.12}$$

$$\text{where } \tau_x = \frac{3}{2}E_x\,, \quad \tau_y = \frac{1}{2}E_y\,.$$

For convenience we have set $\mu_0 = g^{1/2}\nu_0$. The radiation stress terms are evaluated as before, and so finally equation (2.12) becomes

$$\psi_x(\frac{\Omega}{H})_y - \psi_y(\frac{\Omega}{H})_x = \frac{(h^{1/2}E_y)_x}{h^{3/2}} + \mu_0[[h^{\frac{3}{2}}(\frac{\psi_x}{h})_{xx}]_x + [h^{\frac{3}{2}}(\frac{\psi_y}{h})_{xx}]_y]\,. \tag{2.13}$$

As before Ω is defined by

$$\Omega = V_x - U_y = (\frac{\psi_x}{H})_x + (\frac{\psi_y}{H})_y\,. \tag{2.14}$$

As before the wave forcing is given by the expression, that is,

$$E = E_0 \cos\, Ky + F_0 \sin\, Ky + G_0(x), \tag{2.15}$$

so that the wave forcing term in (2.13) again simplifies to

$$(K\cos\, Ky)\frac{(h^{1/2}F_0)_x}{h^{3/2}} - (K\sin\, Ky)\frac{(h^{1/2}E_0)_x}{h^{3/2}}. \tag{2.16}$$

Thus again we observe that the unmodulated term $G_0(x)$ plays no role here at all. In order to match at $x = x_b$ with the expression $Y = \sin\, Ky$ for the streamfunction in the shoaling zone, we should try for a solution of (2.12) of the form

$$\psi = E(x)\cos\, Ky + F(x)\sin Ky + G(x)\,. \tag{2.17}$$

Note that, comparing this with the analogous expression,

$$\psi = F(x)\sin\, Ky + G(x), x < x_b \tag{2.18}$$

in the friction-free case we see that here the term $E(x)$ is purely due to friction. Next, equation (2.17) yields

$$\Omega = \tilde{E}\cos\, Ky + \tilde{F}\sin\, Ky + \tilde{G}, \tag{2.19}$$

where $\tilde{F}$, $\tilde{E}$ and $\tilde{G}$ are the differential operators

$$\tilde{E} = (\frac{E_x}{h})_x - \frac{K^2E}{h} \tag{2.20}$$

$$\tilde{F} = (\frac{F_x}{h})_x - \frac{K^2F}{h} \tag{2.21}$$

$$\tilde{G} = Z_x, \;\; Z = \frac{G_x}{h}\,. \tag{2.22}$$

The left-hand side of equation (2.13) contains terms in $\cos\, 2Ky, \sin\, 2Ky, \cos\, Ky, \sin\, Ky, 1$, while the right-hand side contains only terms in $\cos\, Ky, \sin\, Ky, 1$. Equating the appropriate coefficients on each side we get that

$$F_x\frac{\tilde{F}}{h} - E_x\frac{\tilde{E}}{h} - F(\frac{\tilde{F}}{h})_x + E(\frac{\tilde{E}}{h})_x = 0, \tag{2.23a}$$

$$E_x\frac{\tilde{F}}{h} - F(\frac{\tilde{E}}{h})_x - E(\frac{\tilde{F}}{h})_x + F_x\frac{\tilde{E}}{h} = 0, \tag{2.23b}$$

$$G_x\frac{\tilde{F}}{h} - (\frac{\tilde{G}}{h})_x F = \frac{(h^{1/2}F_0)_x}{h^{3/2}} + \frac{\mu_0}{K}[[h^{\frac{3}{2}}(\frac{E_x}{h})_{xx}]_x - K^2[h^{\frac{3}{2}}(\frac{E}{h})_{xx}]], \tag{2.23c}$$

$$E(\frac{\tilde{G}}{h})_x - G_x\frac{\tilde{E}}{h} = -\frac{(h^{1/2}E_0)_x}{h^{3/2}} + \frac{\mu_0}{K}[[h^{\frac{3}{2}}(\frac{F_x}{h})_{xx}]_x - K^2[h^{\frac{3}{2}}(\frac{F}{h})_{xx}]], \tag{2.23d}$$

$$E(\frac{\tilde{F}}{h})_x - F_x\frac{\tilde{E}}{h} + E_x\frac{\tilde{F}}{h} - F(\frac{\tilde{E}}{h})_x = \frac{2\mu_0}{K}[h^{\frac{3}{2}}(\frac{G_x}{h})_{xx}]_x. \tag{2.23e}$$

The boundary conditions are analogous to those imposed in the friction free case, that is the inviscid regime. That is, at $x = 0$ where $h = 0$ both mass transport fields U, V should vanish, that is from (2.11), $\psi =$ constant and $\psi_x/h = 0$, which implies that

$$\frac{(E,F)}{h} = \frac{(E_x, F_x)}{h} = 0\,, \ G = \text{constant}, \ \frac{G_x}{h} = 0\,, \ \text{at } x = 0\,. \tag{2.24}$$

As before there are also the matching conditions for E, F, G and G separately at the breakerline, that is, we now have that

$$\frac{F_x(x_b)}{F(x_b)} = \frac{X_x(x_b)}{X(x_b)}\,, \quad E_x(x_b) = G_x(x_b) = 0\,. \tag{2.25}$$

Equations (2.23a, 2.23b, 2.23c, 2.23d, 2.23e) form five equations for only three unknowns. Hence in general there is unlikely to be an exact solution, and instead we seek an approximate solution. First, we note that when $\mu_0 = 0$ (the frictionless case) there is an exact solution, as then we can satisfy (2.23c) with $E = 0$, and then provided we also set $E_0 = 0$, equations (2.23d, 2.23e) are also satisfied, leaving only (2.23a) for F and (2.23c) for G. This of course is just the procedure we have followed above. Hence we shall regard the frictional terms as a perturbation on this and so treat ν_0 as a small parameter, noting that ν_0 is dimensionless. Thus we infer that $E = 0(\nu_0)$, and for consistency we must then also choose $E_0 = O(\nu_0)$; indeed we will set $E_0 = 0$ for simplicity.

It now follows that to leading order in ν_0, F is given again by the frictionless solution, so that it satisfies the inviscid solution

$$(\frac{F_x}{h})_x - \frac{K^2F}{h} = ChF \tag{2.26}$$

again. Indeed, since $E = O(\nu_0)$, the error incurred for F is $O(\nu_0^2)$ from (2.23a). Next we see that the frictional term in (2.23c) is $O(\nu_0^2)$, so that also G is again given by the frictionless solution see [17], with an error of $O(\nu_0^2)$. It remains to determine the leading order term for E. For this purpose we can use (2.23e), since the alternative equation (2.23d) generates only a term for E which is $O(\nu_0^2)$. Then, using the above estimates for F, G we see that (2.23e) becomes

$$ChE - \tilde{E} = \frac{2\mu_0}{KF}h^{5/2}Z_{xx}\,, \tag{2.27}$$

$$\text{that is} \quad ChE - (\frac{E_x}{h})_x + \frac{K^2E}{h} = \frac{2\mu_0}{KF}h^{5/2}Z_{xx}\,, \tag{2.28}$$

on using (2.20), where a constant of integration has been set to zero. Here the right-hand side can be regarded as known, and is given by the expression

$$F(CG_x - (\frac{\tilde{G}}{h})_x) = \frac{(h^{1/2}F_0)_x}{h^{3/2}} \tag{2.29}$$

Note that equation (2.29) is from the frictionless case, that is, the inviscid solution for $Z = G_x/h$. Thus we see that the essential outcome of the frictional terms is to introduce into the surf zone an extra component $E(x)\cos Ky$ which, in the longshore direction, is out-of-phase with the component proportional to $\sin Ky$ in the shoaling zone.

2.2 Applications To Quadratic Depth Profile

Now for the quadratic depth profile, where $h = \beta x^2$, in which case F is given by

$$F \approx A_0 x^3 (1 - \frac{x^6}{3x_b^6}) \tag{2.30}$$

and Z is given by

$$Z = \frac{45 g \beta^2 \gamma^2 x_b^2}{16 A_0} (\frac{x^2}{x_b^2} - \frac{1}{\sin[2/\sqrt{3}]} \sin[\frac{2x^3}{\sqrt{3x_b^3}}]). \tag{2.31}$$

Again we note that the equations (29) and (30) are both found in the inviscid solution [20].

We follow the same procedure as for the linear depth profile, in which the right-side is approximate in the limit $x \to 0$. In that case $F \propto x^3$, $Z \propto x^2$ and so the right-hand side is proportional to x^2 that is,

$$\frac{h^{5/2} Z_{xx}}{F} \approx \frac{45 g \gamma^2 \beta^{9/2} x^2}{8 A_0^2} . \tag{2.32}$$

Also the particular solution of (2.28) is proportional x^6,

$$E_p \approx -\frac{5 g^{3/2} \nu_0 \gamma^2 \beta^{11/2} x^6}{8 K A_0^2} . \tag{2.33}$$

The homogeneous equation is

$$E_{xx} - \frac{2}{x} E_x - K^2 E = C h^2 E, \tag{2.34}$$

or, putting $u = x^3$

$$9 E_{uu} - \lambda E - \frac{K^2 E}{u^{4/3}} = 0 , \tag{2.35}$$

where $\lambda = C\beta^2$ as in the inviscid solution. But unlike the linear depth case, there are apparently no solutions available in terms of known special functions. However, again for simplicity we seek just an approximation as $x \to 0$, in which case the solution which satisfies the boundary condition (2.24) is

$$E_h \approx u = x^3 .$$

Finally we need to combine E_h, E_p to satisfy the boundary condition at (2.25) at $x = x_b$ so that

$$E = \frac{5 g^{3/2} \nu_0 \beta^{11/2} x_b^6}{8 K A_0^2} (-\frac{x^6}{x_b^6} + \frac{2x^3}{x_b^3}). \tag{2.36}$$

Also we invoke G from the inviscid solution and is given by

$$G = \frac{45 g \gamma^2 \beta^3 x_b^5}{16 A_0} (\frac{x^5}{5 x_b^5} + \frac{1}{2\sqrt{3} \sin[2/\sqrt{3}]} \cos[\frac{2x^3}{\sqrt{3x_b^3}}] - \frac{1}{5} - \frac{1}{2\sqrt{3} \tan[2/\sqrt{3}]}) . \tag{2.37}$$

Thus we get from (16,35, 29, 36) in $x < x_b$ that the normalized streamfunction ψ_n is again given by, that is,

$$\psi_n = \frac{F(x)}{F(x_b)} \sin Ky + R \frac{G(x)}{G(0)} + S \frac{E(x)}{E(x_b)} \cos Ky , \quad \text{for} \quad 0 < x < x_b , \tag{2.38}$$

where R is now given by

$$R = \frac{135 g \beta^3 \gamma^2 x_b^2}{32 A_0^2} (\frac{1}{2\sqrt{3} \sin[2/\sqrt{3}} - \frac{1}{5} - \frac{1}{2\sqrt{3} \tan[2/\sqrt{3}]}) . \tag{2.39}$$

while

$$S = \frac{15\nu_0 g^{3/2}\beta^{11/2}x_b^3}{16KA_0^3}\,. \tag{2.40}$$

But we recall from (2.39) that

$$R = -\frac{2.1g\beta^3 x_b^2}{A_0^2}\,,$$

and so now we can write

$$S = \tilde{S}|R|^{3/2}\,, \quad \text{where} \quad \tilde{S} = \frac{0.1(\text{sign}A_0)\nu_0\beta}{K}\,. \tag{2.41}$$

Note that it increases with β but decreases with K. Using $Kx_b = 0.2$ as in the inviscid regime, and again estimating $\nu_0\beta x_b \approx 0.01$, we infer that a suitable value is $\tilde{S} = 0.005$, which is much smaller than for the case of the linear depth profile.

The normalized streamfunctions (37) are again plotted for the same values of $R = -0.02$, $R = -0.1$, $R = -0.5$ and $R = -2$ used in the inviscid regime respectively. In contrast to the figures in the invsicid case, we again see that the rip-currents in the frictional regime [7, 19, 20, 21] are modified. But in contrast to the linear depth profile case, the frictional effect is much less discernible.

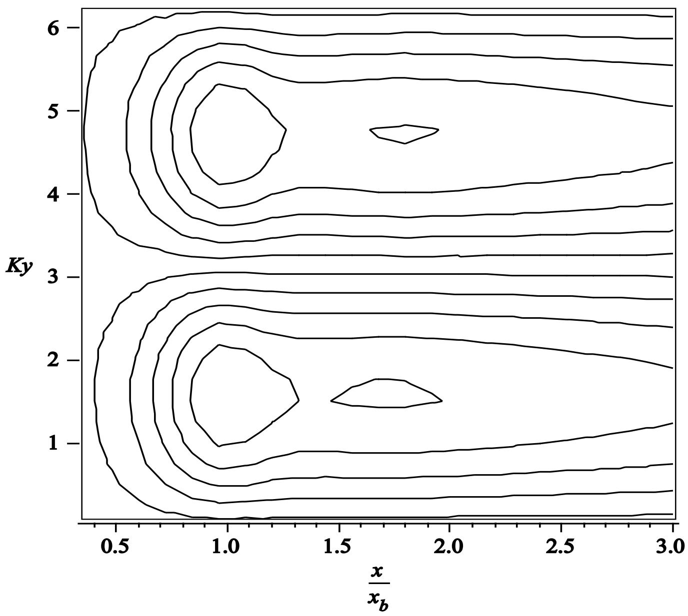

Fig 1. Plot of the rip current streamlines for a quadratic depth profile, given by equation (2.38) where $F(x)$ and $G(x)$ are equations (2.30) and (2.37) respectively for $R = -0.02$ and $\tilde{S} = 0.005$

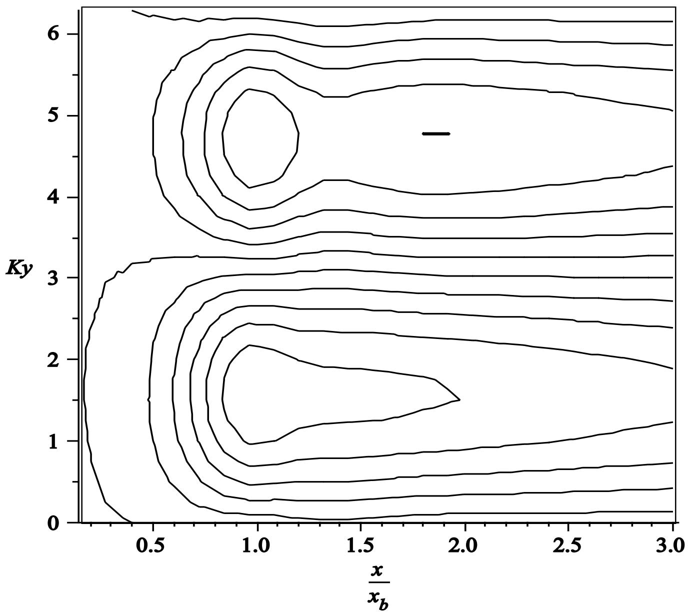

Fig 2. As for figure 2.2 but $R = -0.1$ **and** $\tilde{S} = 0.005$

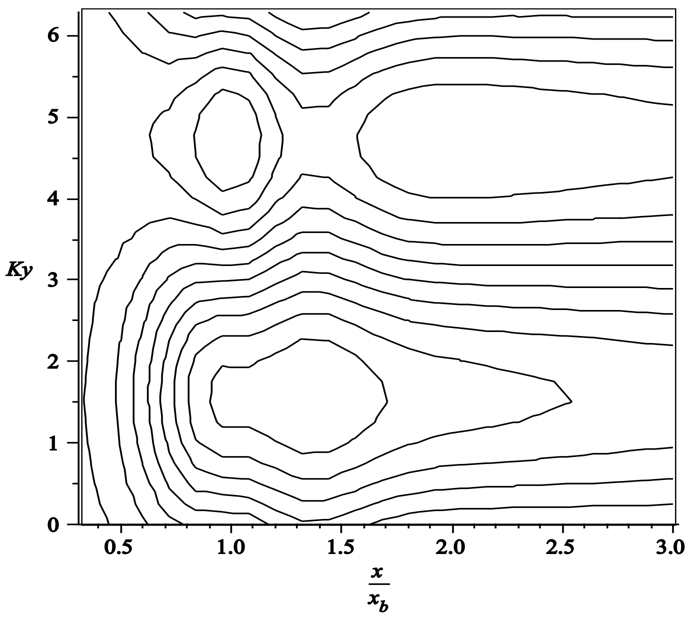

Fig 3. As for figure 2.2 but $R = -0.5$ **and** $\tilde{S} = 0.005$

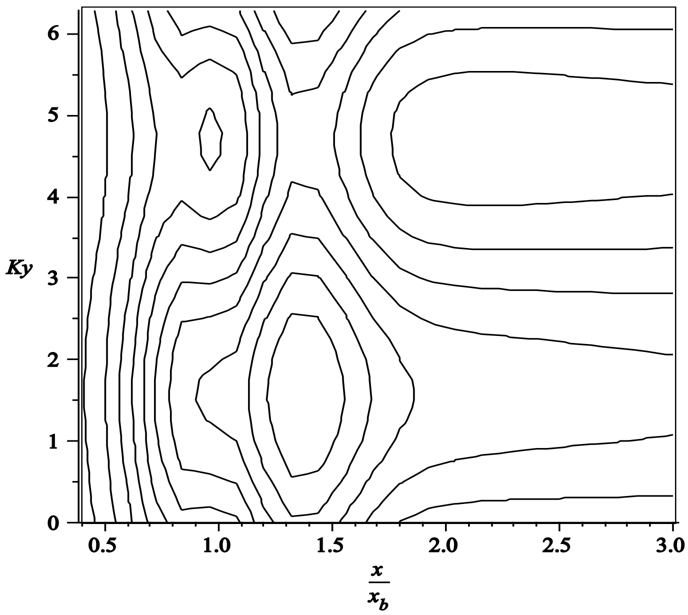

Fig 4. As for figure 2.2 but $R = -2$ **and** $\tilde{S} = 0.005$

3 Conclusion

These flows take place in the shallow surf zone and frictional effects could be expected to be significant. It was nice to see frictionally effects discussed more carefully. However, no new effects appeared but the point can now be made that friction does not destroy the vortex. The underlying problem is of interest and the finding of a free vortex is new and interesting. From the fore-going, it was observed that frictional effects modify rip-currents on a linear depth profile more than on the counterpart quadratic depth profile. Obviously, this can be as a result of the differences in the beach types. As wave forcing is large there is an improved output of the longshore component of the rip-currents . Overall this means that with large R the near-shore current system and its circulation becomes noticeable in the region.

Competing Interests

Author has declared that no competing interests exist.

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